



Algebraic Reasoning Content Academy – Grade 5

Day 2: Multiple Representations
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Adding It Up: Helping Children Learn Mathematics

- Conceptual Understanding: comprehension of concepts, operations and relations
- Procedural Fluency: skill in flexibly, accurately, efficiently, and appropriately using procedures
- Strategic Competence: ability to formulate, represent and solve problems
- Adaptive Reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive Disposition: view of math as useful, sensible and worthwhile coupled with belief in diligence and personal efficacy



Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

Problem Solving—Van De Walle

- The problem must begin where the students are
- The problematic or engaging aspect of the problem must be due to the mathematics
- The problem must require justifications and explanations for answers and methods.

Show & Tell Approach—Van De Walle

Listen to a description from Van de Walle's book of outcomes in a teacher directed approach.

Problem Solving—Van De Walle

- Focuses students' attention on ideas and sense making.
- Develops the belief in students that they are capable of doing mathematics and that mathematics makes sense.
- Provides ongoing assessment data.
- Excellent method for attending to a breadth of abilities.
- Develops “mathematical power.”
- IT IS A LOT OF FUN

Provide Hints but Not Solutions

—Van De Walle

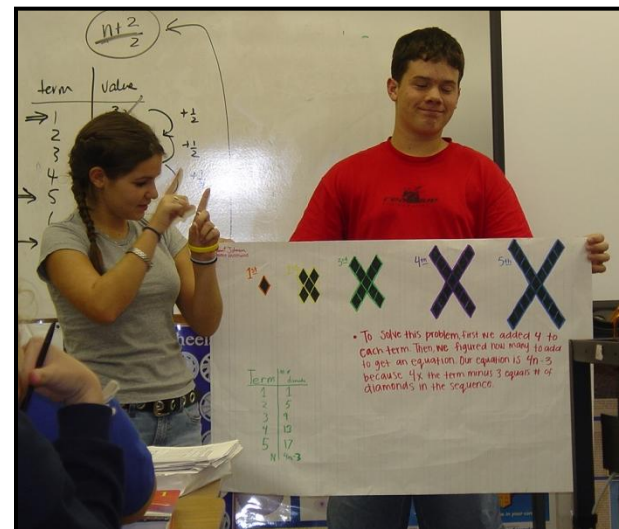
“Always keep in mind that as soon as students sense that you have a method of solving the problems, they will almost certainly stop searching for their own methods because they are convinced that your way must be best.”

Gr 3-5 Teaching Student-Centered Mathematics—Van De Walle pg 21-24

- What can I tell them? Should I tell them anything?
- How will I be able to teach all of the basic skills?
- Why is it okay for a student to “tell” or “explain” but not for me?
- This approach take more time, how will I have time to cover everything?
- Do I need to use a problem-based approach every day?
- Is there any place for drill and practice?
- My textbook is traditional, how can I use it?
- What do I do when a task bombs or students don’t “get it”?

Require students to defend and talk about their solutions because communication deepens understanding.

When did the mathematics make sense for you?



#1 Answer:

When I started teaching it!!!

Day 2 Learning Outcomes

- Use both real-life and abstract representations to solve problems
- Introduce the value of multiple representations: table, picture, equation, graph
- Recognize that constant rate of change is a defining characteristic of linear functions
- Investigate what can be learned from analyzing students' work
- Solve, analyze, and discuss 5th grade algebraic reasoning tasks, connection to MS & HS Algebra Tasks

Bird Watching



- One of Justin's hobbies is bird watching. He was excited because he had moved to a new home in an area that had a large bird population. On the first day he saw 2 birds near his home and on the second day he saw 5 birds near his home. On the third day he saw 8 birds, and on the fourth day he saw 11 birds. If this pattern continued, how many birds would Justin see on the tenth day? How many would he see on the hundredth day?
- Extension: Write a rule for finding how many birds he will see on any number of days.

Analyzing Student Work

- Examine each students' response.
What does it suggest the student understands?
Does not understand? Why?
 - Which response do you think shows the greatest understanding? Why?
 - Which response do you think shows the least understanding? Why?
- Rank your group's student responses from weakest to strongest using a 1, 2, 3, and 4 ranking. 1 being weakest understanding.

Analyzing Student Work

- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

Sample A

I need to find how many b/hds will I just in 100 days.
day. I will make a table

day	birds
1	2
2	5
3	8
4	11
5	14
6	17
7	20
8	23
9	26
10	29
11	32

$11 + 11 + 11 = 33 - 1 = 32$
 $1 \times 3 = 3$
 $2 \times 6 = 12$

on day 100
 $100 + 100 + 100 = 300 - 1 = 299$

Sample B

You have to find how many birds

Birds he saw

days	birds
1	2
2	5
3	8
4	11

2b he saw 2b birds on day 10
he saw 12b birds on day 100

Sample C

I need to find how many birds Justin saw,
 I know that he saw 2 birds on the First
 Birdwatching day and 3 more
 than that.

I noticed that they
 are counting
 by 3's


days	birds
1	2
2	5
3	8
4	11
5	14
6	17
7	20
8	23
9	26
10	29

$3 \times d - 1 = b$

key
 d is day
 b is bird

$100 \times 3 - 1 = 299$

$9 \times 3 - 1 = 26$
 $3 \times 9 - 1 = 26$



Sample D

I have to tell you the birds he
saw on day 10 and 100.

1-2
2-5
3-8 birds
day 4-11 birds
5-14
6-16
7-19
8-22
9-25
10-28 ←

100 day is 128

Analyzing Student Work

- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

Growing Triangles



Figure 1

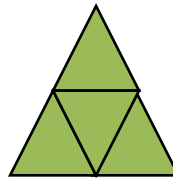


Figure 2

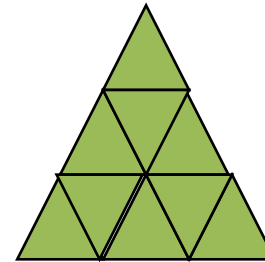


Figure 3

- Using pattern blocks, construct the 3 figures above.
- Describe the pattern you notice in the figures.

Growing Triangles



Figure 1

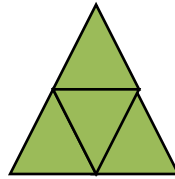


Figure 2

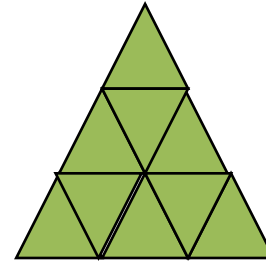


Figure 3

- How many triangles are needed to create the next figure?
- How many triangles are needed to create the 10th figure? 15th figure? 25th figure?
- Write a rule representing the relationship between the figure number, f , and the number of triangles needed to make the figure, n .

Growing Triangles

- Create your own growing pattern using any pattern blocks of the SAME shape. Draw the first three figures.
- What is the algebraic rule for your growing pattern between the figure number and the number of triangles needed to construct the figure?

Tile Patterns



Tiling around a Fountain

- **SCENARIO:** You are working on a landscaping crew for the county parks department this summer.
- To avoid the mud that surrounded the park fountains last summer, your crew is planning to put a border of tiles around each of the square fountains in the park. The border tiles each measure 1-foot on each side. Your foreman shows you this diagram for the smallest fountain. You notice that a fountain that has a base of 1 square foot will require 8 border tiles.
- Using this pattern, how many tiles will be needed for different size square fountains?



Tiling around a Fountain

Problem 1: If a square fountain has sides of length s feet, how many tiles are needed to form the border?

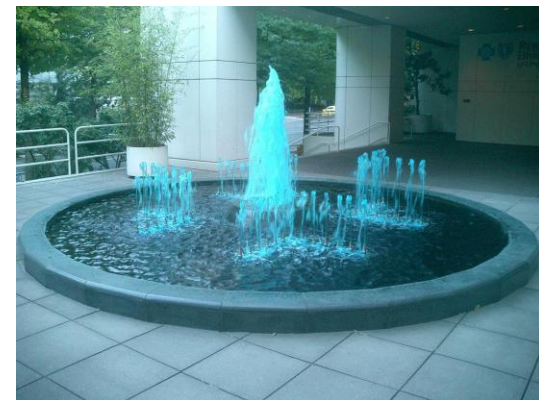
- Using grid paper draw a diagram of the designs for the border of fountains with side lengths of 2, 3, 4, 6 and 10 feet. Record your results in a table.
- Write an equation for the number of tiles, N , needed to form a border for a square fountain with side length of S feet.
- Generate as many equations as you can for this relationship.
 - Are the equations the same?
 - How can you convince someone that your expressions for the number of tiles needed are equivalent?



Group Poster

Tiling around a Fountain

- Show all the expressions generated by your group
- Explicitly connect your expressions to the diagrams/concrete model



Gallery Walk

Tiling around a Fountain

- One person from each group “mans” the group’s poster to answer questions.
- Rest of the group members view other posters. Look for:
 - The most common representations
 - The most unique representations



Re-Tiling

Tiling around a Fountain

Follow-up Problem: Make a table and a graph for each equation you found in problem 1.

- Do the tables and graphs show that the expressions and equations are equivalent?
- Is this relationship between s and n a straight line, or a curve?



Collection of Expressions

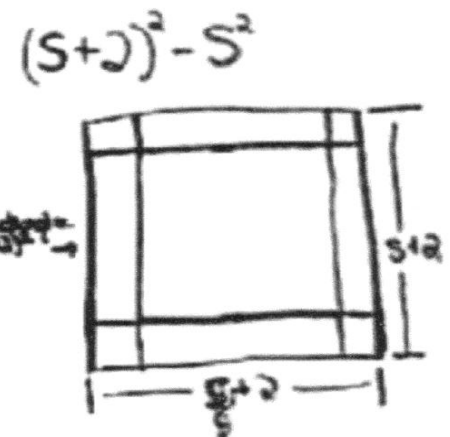
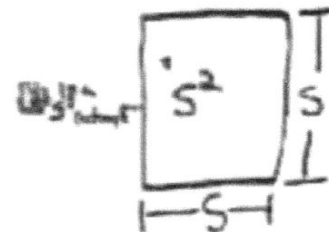
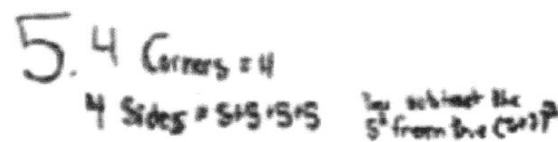
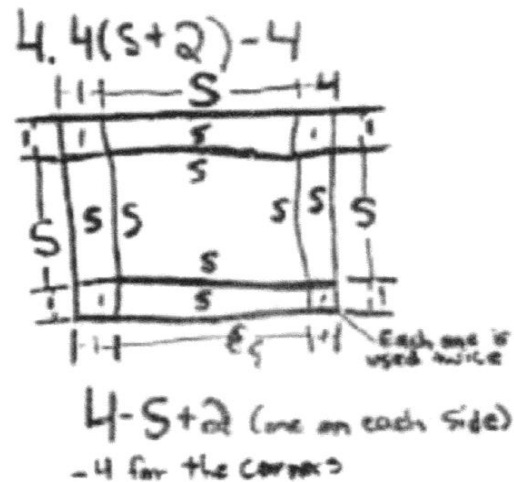
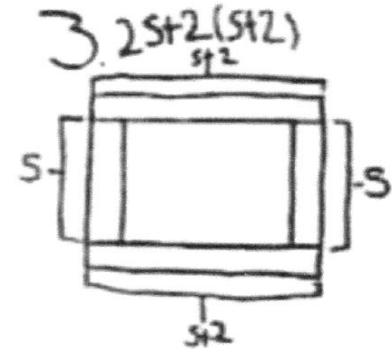
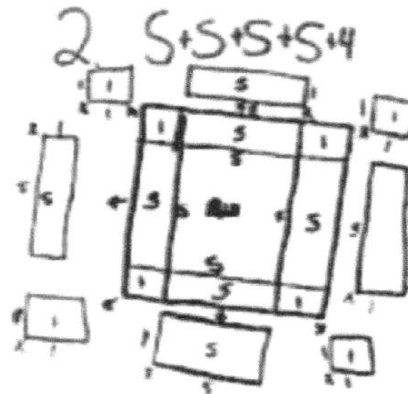
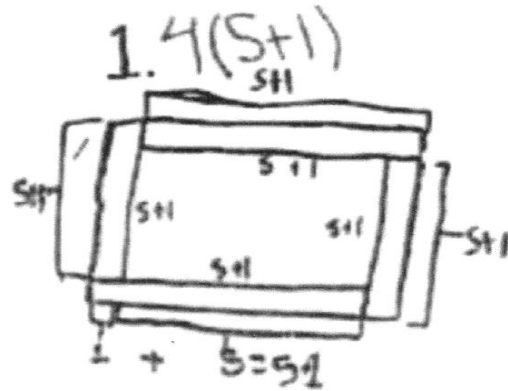
Tiling around a Fountain

- Make a collective list of all the expressions that were generated in the groups.



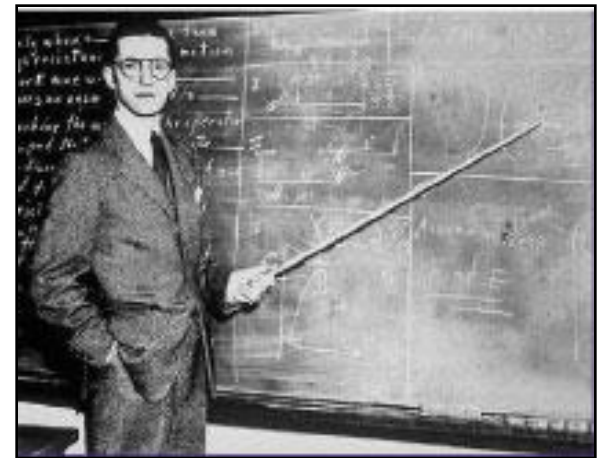
One Group's Work

Tiling around a Fountain



Looking through Teacher Lenses

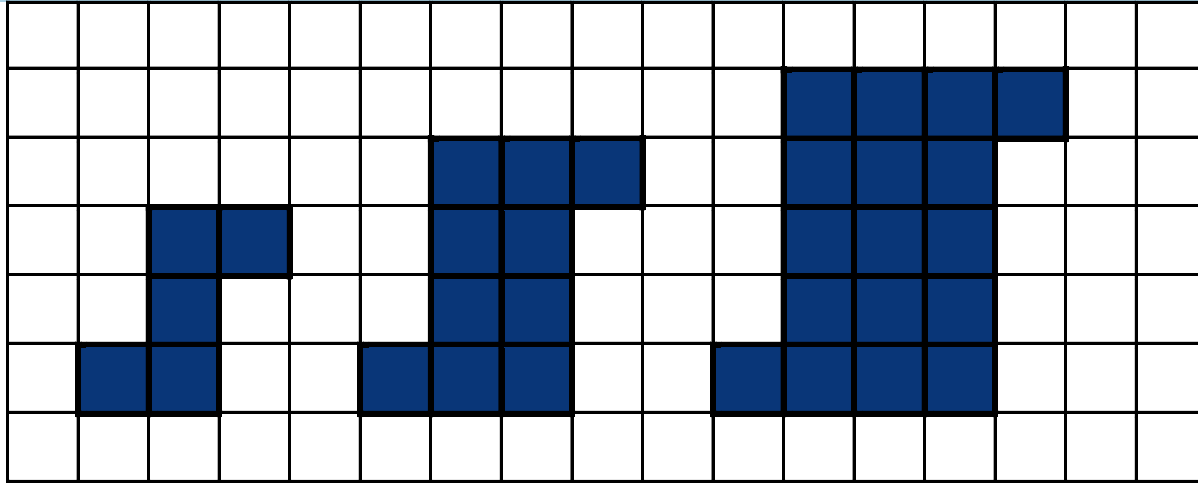
- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



A Different Tiling Pattern



The S-Pattern Task



- Find the number of tiles for the next two figures in the pattern.
- Write as many different equations as you can for the number of tiles needed for any figure in the pattern.

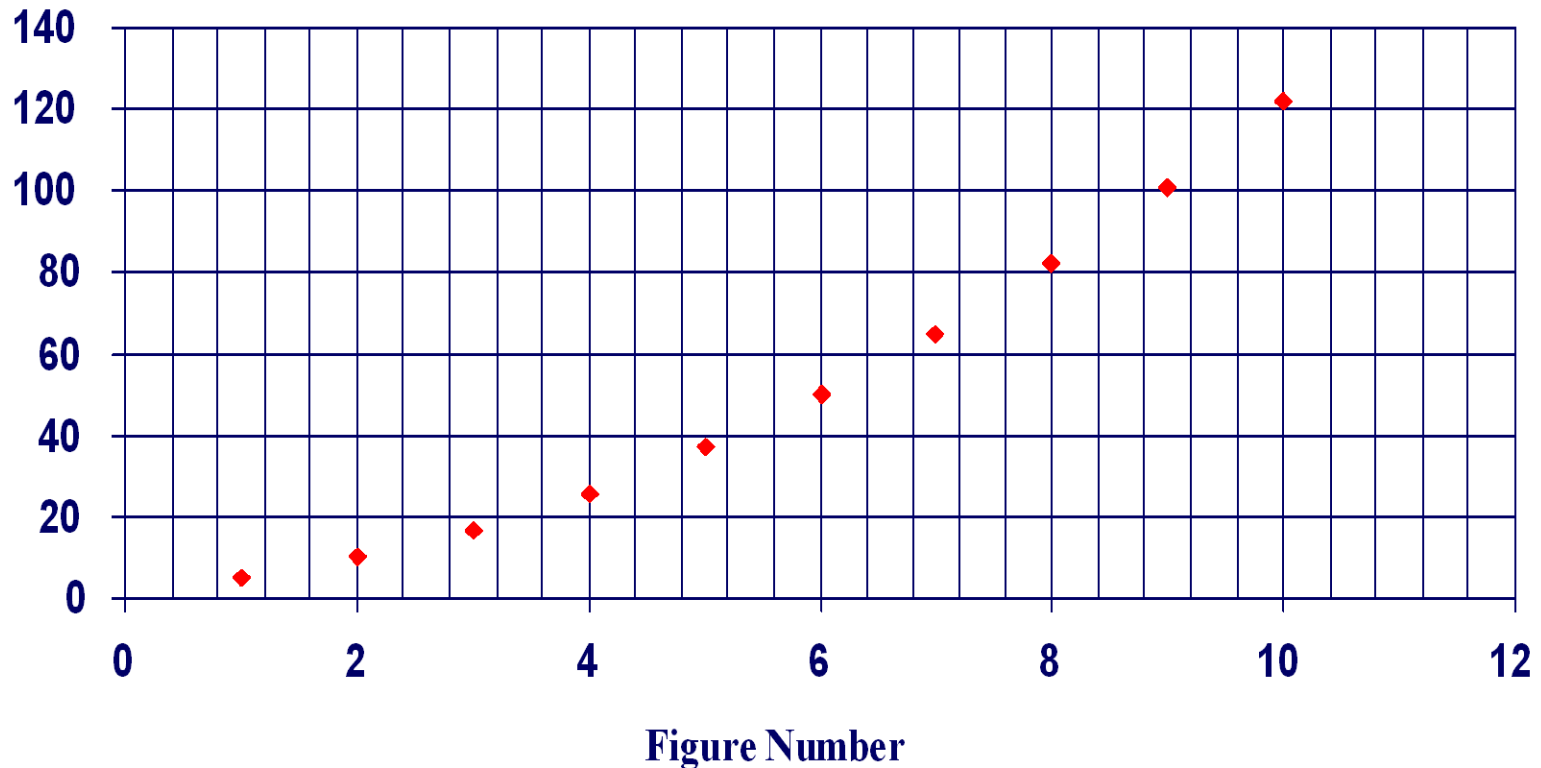
The S-Pattern Task

- Make a graph that shows the relationship between the figure number and the number of tiles in the figure.
- Share your solutions with your group.



A Graph of the Pattern

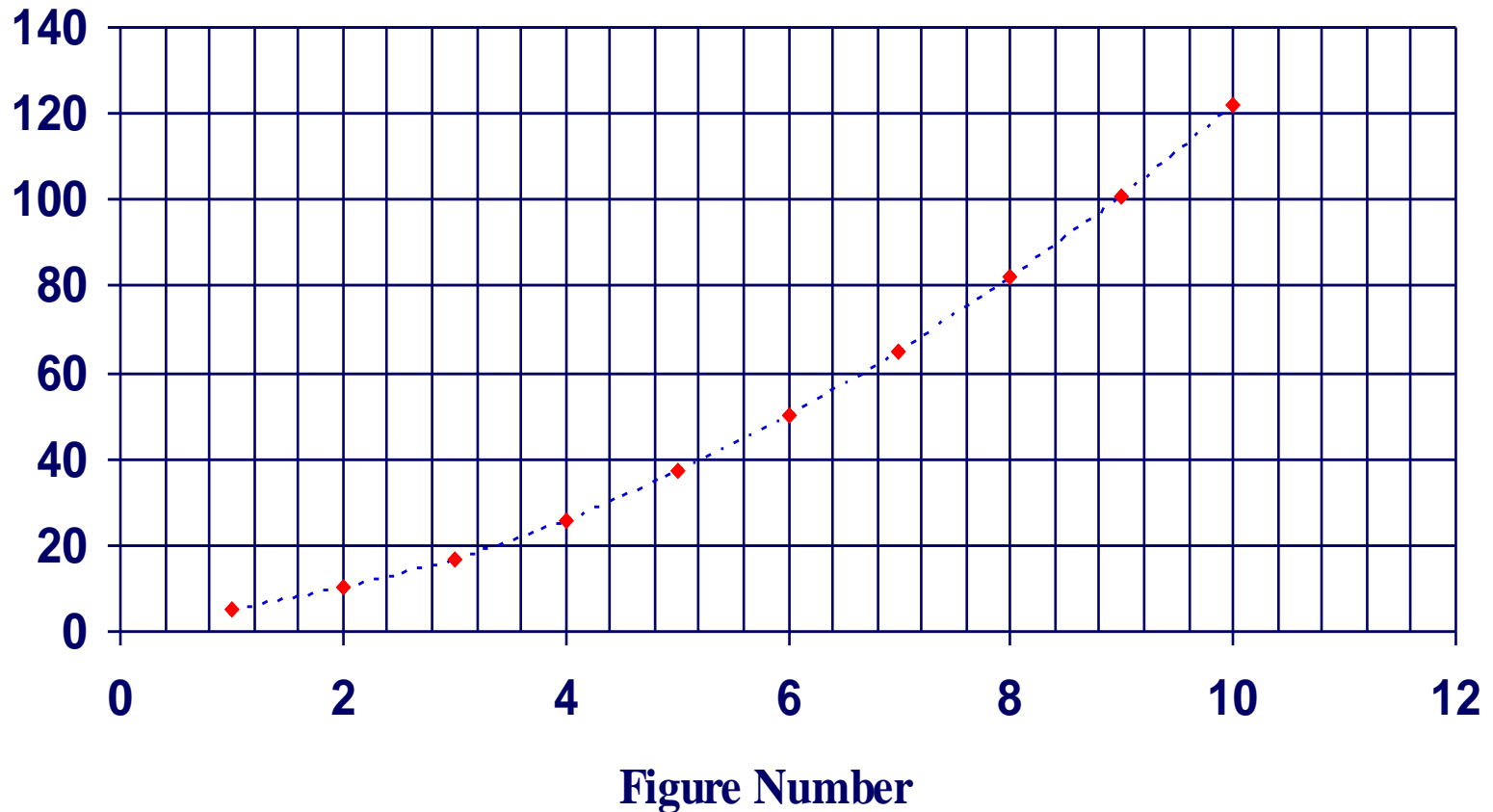
The S-Pattern Task



How does the S-Pattern graph compare to the Tiling the Fountain graph?

Another Graph of the Pattern

The S-Pattern Task



A Graph of All Values

The S-Pattern Task

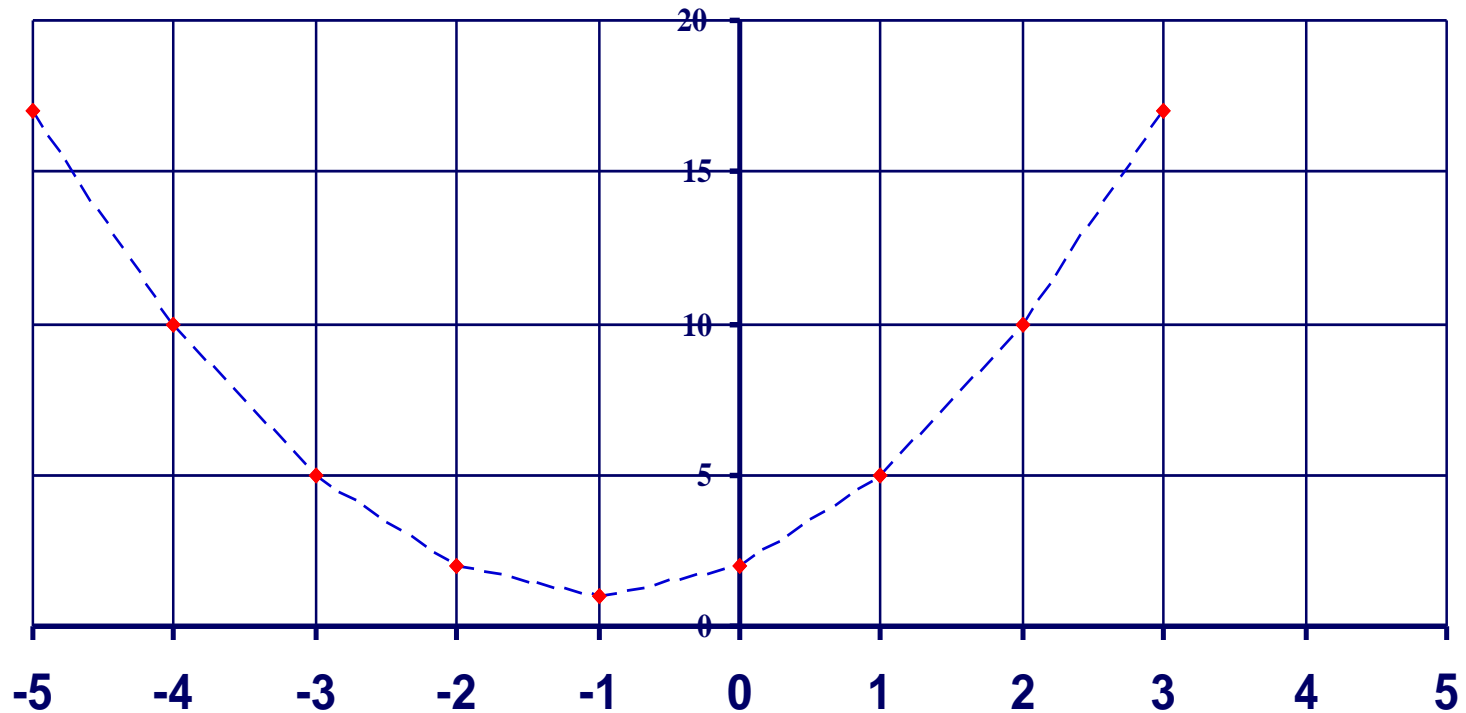


Figure Number

De-Briefing the S-Pattern Task

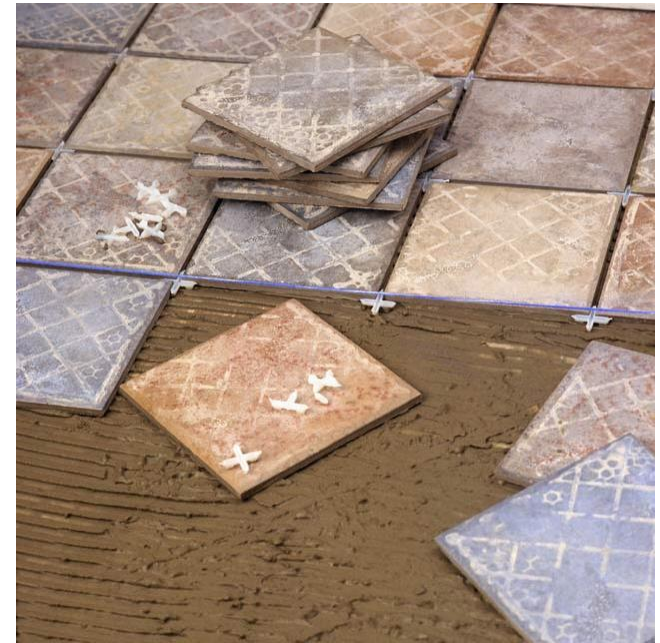
- Contrast the strategies that were presented.

What were the similarities and what were the differences?



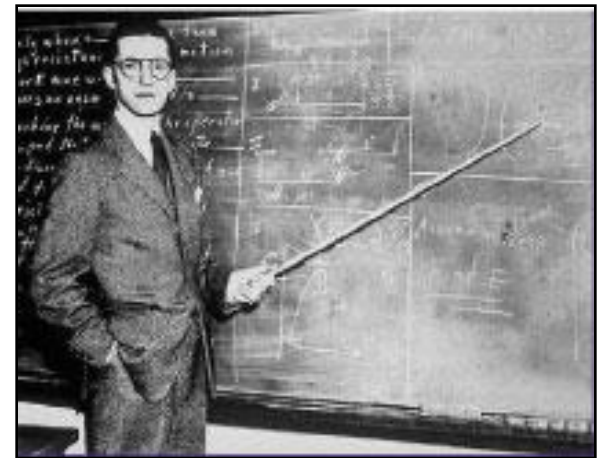
Comparing the Two Tasks

- How are the patterns in tiling and S-pattern the same?
- How are the patterns different?



Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
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Why do we want kids to experience Pattern Tasks?

- Understanding variables
- Connecting between representations
- Progressing on standards
- Using context for working on functions
- Starting the problem at entry points
- Finding and comparing solutions



Analyzing Student Work

The S-Pattern Task

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Telephone Task

Scenario: A cellular telephone company charges \$10.00 per month plus \$0.20 per call. Complete the table and graph the results.

Number of Calls	0	1	2	3	4	5	10	20
Total Cost (Dollars)								

Write an algebraic rule to determine the total cost, c , when given the number of calls, n .

Model Car Racing Task

The distance formula $d = 0.25t$ represents the distance, d , a model racing car travels in t minutes at a rate of 0.25 kilometers per minute.

Labels	Time	Distance
Unit	Minutes	Kilometers

Model Car Racing Task

Labels	Time	Distance for Model Car #1	Distance for Model Car #2
	Minutes	Kilometers	Kilometers
Unit	t	$0.25t$	$0.5t$

- As the time traveled increases, what happens to the distance traveled?
- How do the values for the distance traveled of Model Car #2 compare to the distance traveled by Model Car #1?
- How is the graph of the 2nd car different from the graph of the 1st car?

U.S. Shirts

Using Tables, Graphs, and Equations

This past summer you were hired to work at a custom T-shirt shop, U.S. Shirts. One of your responsibilities is to find the total cost of customers' orders. The shop charges \$8 per shirt with a one-time set-up fee of \$15.

1. What is the total cost of an order for 10 shirts?
2. What is the total cost of an order for 100 shirts?
3. Explain how you found the total costs.
4. How many T-shirts can a customer buy for \$60?
5. How many T-shirts can a customer buy for \$250?
6. Explain how you found the number of shirts that can be purchased.



U.S. Shirts

Using Tables, Graphs, and Equations

Make a table of values for the problem situation.

Labels	Number of Shirts Ordered	Total cost
	Shirts	\$
Unit		



U.S. Shirts

Using Tables, Graphs, and Equations

- Create a graph of the data from your table. First, choose your bounds and intervals. Remember to label your graph clearly and add a title to the graph.
- Use your graph to determine the price of 40 shirts and 27 shirts. Use your graph to determine how many shirts can be purchased for \$300 and for \$540.



U.S. Shirts

Using Tables, Graphs, and Equations

- Write an algebraic equation for the problem situation.
- In this lesson, you have represented the problem situation in four different ways: as a sentence, as a table, as a graph, and as an equation. Explain the advantages and disadvantages of each representation.



Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
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